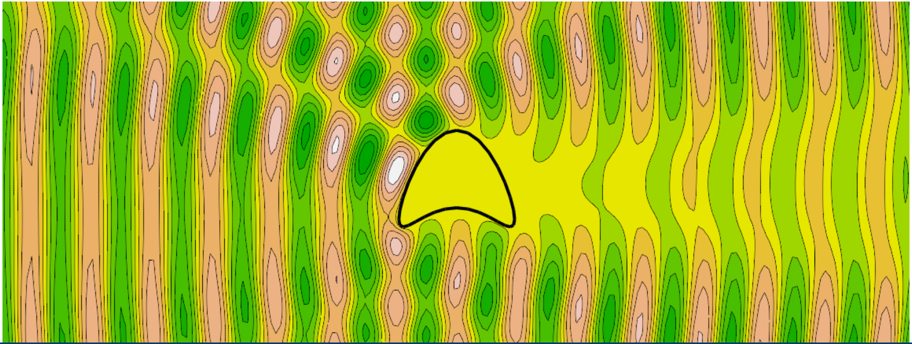


A SCIENTIFIC USE CASE

Direct and inverse acoustic scattering

MATLAB for HPC | November 6, 2023 | Andreas Kleefeld | Jülich Supercomputing Centre, Germany

JÜLICH SUPERCOMPUTING CENTRE

MOTIVATION

A simple example

Time-harmonic acoustic (electromagnetic/elastic) waves can be used to

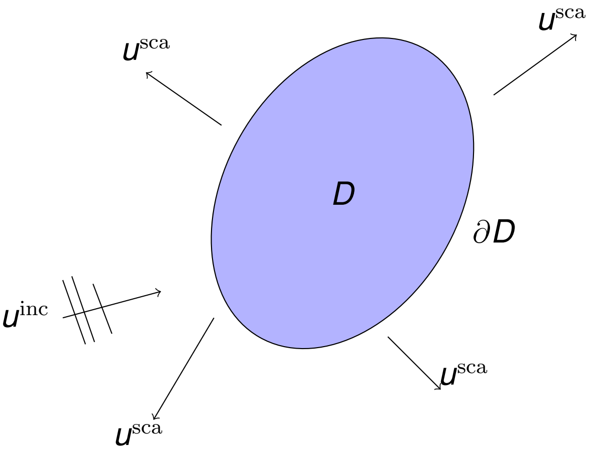
- detect defects in material (non-destructive testing).
- visualize the interior of a body (medical imaging).
- locate buried object.

$\Longrightarrow$ This is an inverse problem.

THE DIRECT PROBLEM

Setup

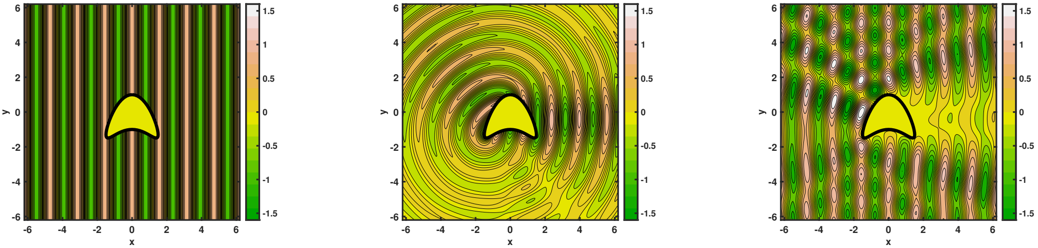
- Given an obstacle D (precisely its boundary ∂D) and an incident field, compute the scattered field (or the far-field) satisfying the Sommerfeld radiation condition.
- A PDE has to be solved.
- Catch: Computational domain is unbounded.
- Solution: Boundary integral equations (BEM).



THE DIRECT PROBLEM

Computation and visualization with MATLAB

Rapid prototyping and easy visualization possible with MATLAB.



(a) Plane incident field (b) Scattered field (c) Total field

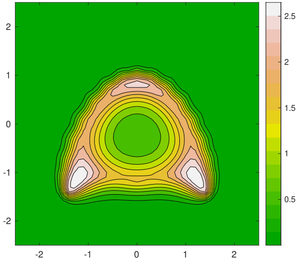
Figure: Real-part of incident field, numerical solution of the acoustic scattering problem, and total field for kite-shaped domain.

THE INVERSE PROBLEM

Synthetic data

Given one incident field and the corresponding far-field (or more than one), find D (or precisely its boundary ∂D)

$\implies$ Inverse problem solved with the factorization method.



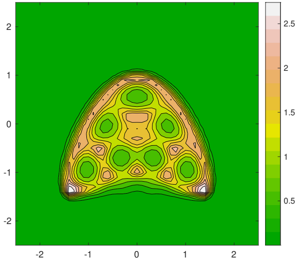


Figure: Reconstructions of a kite-shaped domain with the factorization method using far-field data with 72 incident and 72 observation directions for the wave numbers 2 and 5.

THE INVERSE PROBLEM

Real measurements

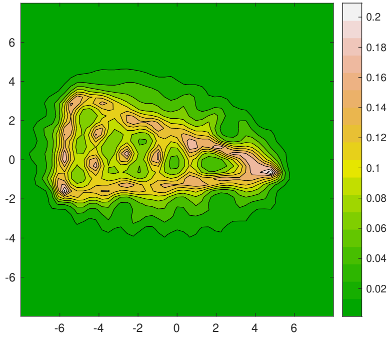


Figure: Reconstruction of an aluminum triangle with the factorization method using far-field data (36 incident and 36 observation directions) for the wave number ≈ 2.0944.

SUMMARY AND OUTLOOK

- PDE solver needs only a few line of code.
- The same is true for the inverse solver.
- Extension to three dimensions and the vector-valued case is straightforward (possible memory issues).
- Code uses ‘basic’ linear algebra.
- BEM is a local method which can be easily parallelized.
- The same is true for the factorization method.

CODE EXAMPLE

The factorization method (the basic one without any regularization)

```
function W=FM(A,wavenumber)
    M=size(A,1);
    ii=0:1:M-1;ti=2*pi*ii/M;
    d=[cos(ti)' sin(ti)'];
    [~,sigma,V]=svd(A);
    diagsigma=diag(sigma);
    N=51;
    grid=linspace(-2.5,2.5,N);
    W=zeros(N,N);
    for i=1:N
        y=grid(i);
        for j=1:N
            x=grid(j);
            rz=exp(-1i*wavenumber*d*[x;y]);
            rhoz=V'*rz;
            W(ii,jj)=1/sum(abs(rhoz).^2./abs(diagsigma));
        end
    end
end
```

Listing 1: MATLAB code for the factorization method.